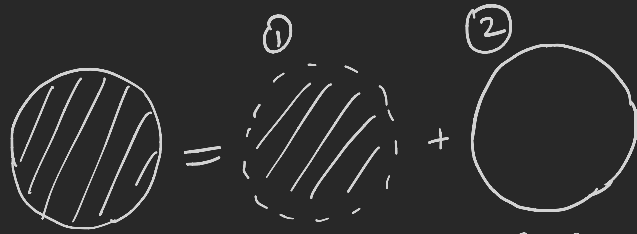
§14.7 #37:

Find extrema of $f(x,y) = 2x^3 + y^4$

on $D = \{(x,y) \mid x^2+y^2 \le 1\}$

D:



$x^2+y^2 \le 1 \quad = \quad x^2+y^2<1 \quad + \quad x^2+y^2=1$

I. Decompose region.

II. Identify candidates for extrema in each region

1. candidates = critical pts of f
   $\nabla f = \vec{0}$.

2. Use Lagrange mult §14.8 or eliminate variable(s).

III. Compare the value of $f(x,y)$ at each candidate pt. from II.

Candidates in 1

$$\nabla f(x,y) = \langle 6x^2, 4y^3 \rangle = \vec{0}$$

Only solution is $(x,y) = (0,0)$.

This does satisfy $x^2+y^2<1$ ✓

Candidates in 2 $(x^2+y^2=1)$

$$2x^3+y^4 = 2x^3+(1-x^2)^2$$

$$= 2x^3+1-2x^2+x^4$$

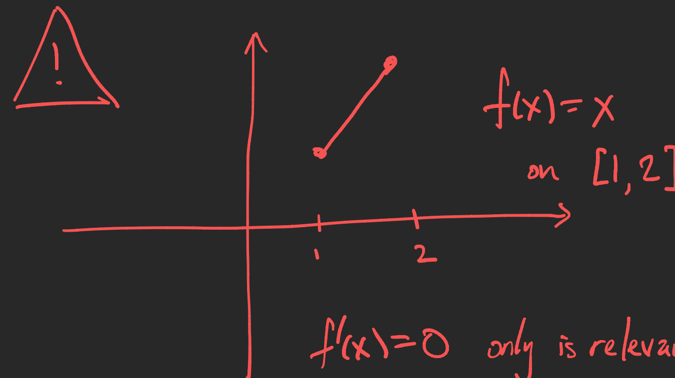
$$g(x) = x^4+2x^3-2x^2+1.$$

$$g'(x) = 4x^3+6x^2-4x = 0$$

$$x(2x^2+3x-2) = 0$$

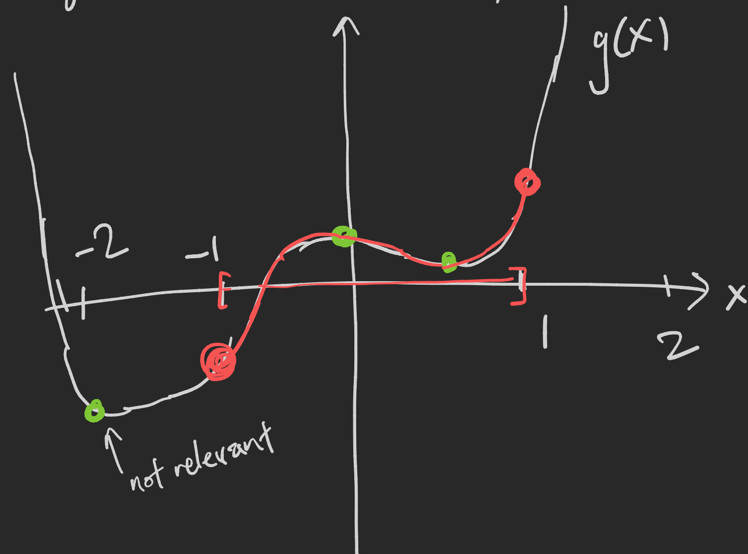
$$x(2x-1)(x+2) = 0$$

$x = 0,\ 1/2,$ or -2.



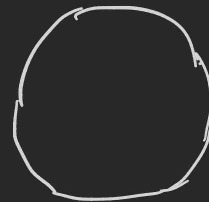
$f'(x)=0$ only is relevant for x strictly between $1, 2$. Does not find extrema in this case.

Why is ⚠ relevant to our problem?



We are not optimizing $g(x)$ on all of $\mathbb{R}$: We are optimizing it on the closed interval $[-1,1]$.

Another way of putting it:



$$x^2+y^2=1$$

. . . . . after eliminating y . . . .

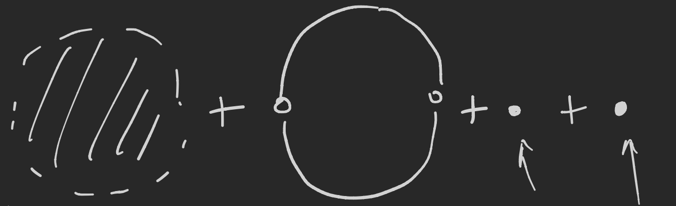
new "boundary" appeared!

$$-1 \le x \le 1$$

$$\|$$

these are candidates too!!!

$-1 < x < 1$ $\quad$ $x=-1$ $\quad$ $x=1$



Candidates: $\nabla f = \vec{0}$ $g'(x) = 0$ $x=-1$ $x=1$

Arclength comment:

$x = \cos t$ $y = \sin t.$ $-\infty < t < \infty$

$$\int_{0}^{2\pi} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt = \cdots$$

need to first figure out what t bounds trace out curve exactly once.

Compute $\lim\limits_{(x,y)\to(0,0)} \dfrac{xy\cos(y)}{3x^2+y^2}$ **if it exists (or write "DNE" if not).**

Compute $\lim\limits_{(x,y)\to(0,0)} \dfrac{\sin^2(\sqrt{x^2+y^2})}{x^2+y^2}$ **if it exists (or write"DNE" if not).**

#1) Try along $y=mx$:

$$\lim_{x\to 0} \frac{m\cancel{x^2}\cos(mx)}{3\cancel{x^2}+m^2\cancel{x^2}} = \frac{m}{3+m^2}.$$

This depends on m, so the limit DNE.

e.g. along $y=0$ get 0, along $y=x$ get $1/4$ and $0 \neq 1/4$.

#2)

$$\lim_{\substack{r\to 0^+\\ \theta \text{ any}}} \frac{\sin^2(r)}{r^2}$$

This is just a SVC limit.

$$= \left(\lim_{\substack{r\to 0^+\\ \theta \text{ any}}} \frac{\sin(r)}{r}\right)^2 = 1^2 = 1.$$